VISUALISATION AND THE DEVELOPMENT OF MENTAL COMPUTATION

JANETTE BOBIS
University of Sydney

The purpose of this paper is to report findings of a study intended to investigate the effect of instructional activities designed to facilitate the development of visualisation strategies in young children. Utilising this intuitive capacity can provide a basis for developing number relations and devising strategies to learn basic facts. Given practice children can develop a reasonably large collection of set dot patterns that they recognise without counting. These patterns begin to be related mentally to one another as they enhance part-part-whole relations. Organising the dots into recognisable subgroups and patterns facilitates the ability to recognise and give number names to groupings, especially when the groupings are larger than five.

Instructional activities and materials that were used to facilitate the development of visualisation strategies in two kindergarten classes over a one year period are described. Students' verbal and pictorial responses are analysed and the implications for fostering the development of basic facts in this manner are discussed.

The results indicated that, with practice, children can become capable of mentally combining and separating patterns, thus able to instantly recognise the whole and its related parts - a very efficient strategy for aiding the recall of basic addition and subtraction facts.

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Should teachers consciously try to develop children's abilities to compute mentally before the time of independent development? From personal experience and that of teachers working with me in recent years, the advice is that developmentally appropriate experiences can and should be offered in the kindergarten and Year 1 mathematics programs. This view is one that other researchers have expressed (Fuson & Secada, 1986; Thornton, 1989) and one that expounds Vygotsky's zone of proximal development (1978). In this zone children can be helped by more capable peers and adults to learn new concepts and skills related to those already possessed. Work with children in their first year of formal schooling has provided evidence of the value of emphasising visualising skills. The aim is to increase the likelihood that prerequisite skills, such as subitisation, are well established before children are asked to apply them for obtaining quick responses to addition and subtraction facts.

The purpose of this paper is to report findings of a study intended to investigate the effect of instructional activities designed to facilitate the development of visualisation strategies in young children. Students' responses revealed that the visualisation activities used fostered the development of skills that enhanced learning of their basic facts.

Visualising number patterns

The process of instantaneous recognition of number patterns without counting is known as subitising (from the Italian word subito - immediately) (Folk et al., 1988). Piaget found that children, by the age of four, are capable of instantaneously recognising groups of one, two, three and four objects. Having been surrounded by groups of objects from a young age, children develop stable mental images of these patterns and they are soon able to reconstruct them for themselves. Constant exposure to the number names used to describe the arrangements help children form pattern-name associations (though they may not fully understand the quantitative aspects of the patterns till later). Utilising this intuitive capacity can provide a basis for developing number relations and devising strategies to learn basic facts. For example, given practice with dot patterns children can develop a reasonably large collection of set patterns that they recognise without counting. These patterns begin to be related mentally to one another as they enhance part-part-whole relations. Organising the dots into recognisable subgroups and patterns facilitates the ability to recognise and give number names to groupings, especially when the groupings are larger than five. Joining and separating visual patterns provides a basis for developing future number

relations. For example, a child might recognise a pattern for "two" and a pattern for "three" in the "five" pattern (see Figure 1).

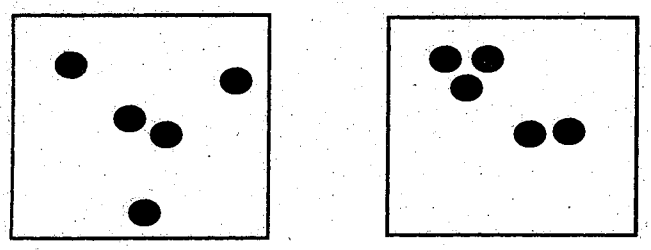


Figure 1 Two dot patterns for the number five. The second pattern is easier to recognise because the dots are arranged into smaller subgroups that facilitate subitisation.

THE STUDY

The following section describes instructional activities and materials that were used to facilitate the development of visualisation strategies in two kindergarten classes over a one year period. Students' verbal and pictorial responses will be analysed and the implications for fostering the development of basic addition and subtraction facts in this manner will be presented.

Participants

Two mixed ability kindergarten classes from a metropolitan Sydney school participated in the study. The classes contained 32 and 35 children that ranged between the ages of 4 years 5 months and 5 years 7 months at the start of first term. Boys and girls were equally represented in the former class but the girls slightly outnumbered the boys in the latter. Two female classroom teachers (one of whom was the chief investigator), participated in the study. Both teachers had more than 5 years teaching experience on a variety of grades. The teachers held meetings almost daily during the initial stages of the study and fortnightly during second, third and fourth terms.

Instructional Materials and Activities

Dot Patterns

A set of dot cards was made that represented the numbers from one to ten as an arrangement of dots. Except for the numbers one, two and three, each number was represented by at least three different dot arrangements. Figure 2 shows the cards that were presented for the number six.

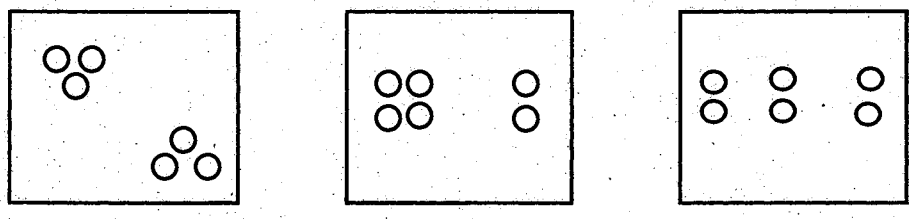


Figure 2 Dot cards for six

The exact size and construction of the cards are not significant. The important aspect of the cards is that they utilise the ability of children to subitise and they encourage mental shifts from thinking about a

number as one type of dot arrangement (or part-part-whole combination) to another. For example, the arrangement for the number five could be seen as a group of three dots and a group of two dots or as a group of four dots with one left over.

Activities with the cards were done with the whole class and with small groups of children working independently. Some of the activities included 'flash-and-tell' and 'find a pair' (see Van de Walle, 1988, for a description of these and many others).

The Ten Frame

Ten frames are two-by-five rectangles into which counters or dots are placed to illustrate numbers less than or equal to ten. This model for numbers was originally designed by Wirtz and is explained in detail by Labinowicz (1985) and Thompson and Van de Walle (1984). Like the dot patterns, the ten frame utilises the developing capacity of young children to subitise. Children learn to recognise patterns and their number names according to the number of dots and empty frames occurring on a flash card. For example, if the ten frame for eight was flashed, the child could be asked:

* How many dots were shown on the card?
* How many empty spaces?
* How many if we add one more (or less) to the number shown?
* How many more to make ten?
* How many less to make five (or any other number)?

Other activities encouraged children to manipulate their mental images by adding or taking away dots to make a new number and then explain how they got their new number to a classmate using drawings, concrete materials and verbal descriptions. The overhead projector was used extensively by the classroom teachers to model these procedures to the class. Once one arrangement for a number was given, children were immediately challenged to think of another way to represent the same number. For a more detailed explanation of the activities see Bobis (1992). (For further activities see: Hoelter, 1987; Labinowicz, 1985; Thornton, 1989; Van de Walle, 1988.)

Activities emphasising visualising skills were integrated into various aspects of the ongoing mathematics programs and were incorporated into short sponge activities throughout the day both during and distinct from the regular mathematics lessons. Starting from week one of the first term in the school year we carried out activities like those described in the previous section and were supplemented by auditory patterning, counting back and counting on activities. Children were encouraged to verbalise the visual patterns they saw for different numbers and discussions concerning 'nice' (or easy to visualise) patterns were regular occurrences. The emphasis was initially on subitising skills using the dot cards for numbers less than five and was then extended to include larger numbers and activities with the ten frames. All the activities focused on identifying groups of numbers rather than counting one-by-one. They also required the children to focus on part-part-whole relationships of various number combinations - especially the decomposition of ten.

Students' Responses

The primary mode of obtaining information about pupils mental images was through their verbal comments in class discussion and individual interviews held in conjunction with their drawings and the instructional materials described above.

Whole class sessions (lasting for about 10 minutes) and individual interviews (lasting for 3 to 5 minutes) were regularly taped and transcribed so as to keep a record of the children's responses. In class discussions children's descriptions of the visual images they developed for the specific groupings varied widely. They formed mental images based on their own particular conceptualisation of part-part-whole relationships. Further, it was observed that as the pupils listened to each others' explanations, they began to think about the visually presented arrangements in more than one way and began to independently elaborate on and extend their own ideas. The following excerpts illustrate the mental strategies children were using to determine the number of dots on ten frames as they were flashed before them.

Excerpt 1:

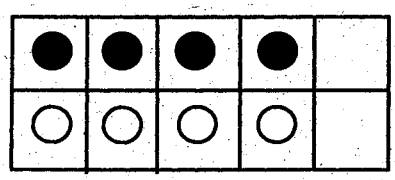
Teacher: (Flashing a ten frame) How many dots did you see?
Lara.

Lara: Nine.
Teacher: How do you know there are nine? You didn't have time to count all those dots.
Lara: There is one empty space and there are ten spaces altogether.
Teacher: How many more dots do we need to make ten dots?
Lara: One more.
Scott: How does she know that? How does she know that there are ten spaces altogether?
Teacher: Lara?
Lara: Because I counted them this morning when I was playing 'Flash' with Tess.
Teacher: Scott, you come and count them.....

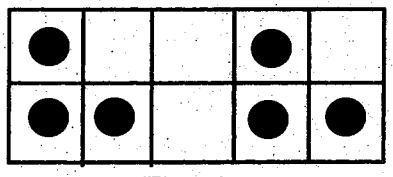
Excerpt 2:
John-Paul: I know why that's a pattern for eight.
Teacher: Come and show everyone then.
(John-Paul moves to front and points to the ten frame.)
John-Paul: See, there's four there and four there and that makes eight.
Teacher: Very good John-Paul.
John-Paul: I know another one.
Teacher: We'd love to hear another.
John-Paul: There's two spaces (pointing to the ten frame).
Teacher: How does two spaces tell you that there are eight dots?
John-Paul: (Pause)...Because..(Pause)..if there were two more there'd be ten.

Although neither John-Paul nor Lara had received any formal instruction in basic addition or subtraction facts, they were already thinking of ten in terms of its parts.

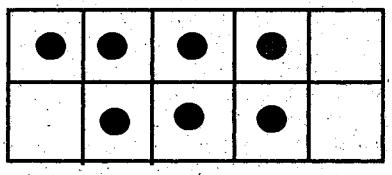
Although we were able to gain insights into students' mental activity through their verbal responses, their drawings also gave indications of how they thought of the visual patterns. The following figures are examples of drawings made by children to record the picture in their mind of their special number. Comments accompanying the drawings were made after reading interview transcripts. Light and dark shades indicate that children used different coloured dots.



Tara

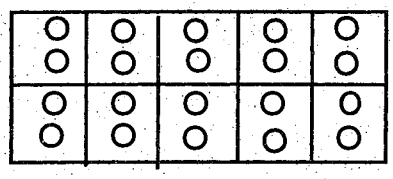


Timothy



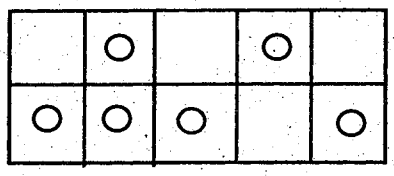
Dominica

Tara, Dominica and Timothy used the visual arrangement of dots to help them represent the mental image of their number. Timothy decided to use a doubling pattern (3 and 3) while Tara used colour to distinguish her pattern of 4 and 4. She decided to draw this number on her ten frame because "I see two spaces and I know there must be 8 dots".

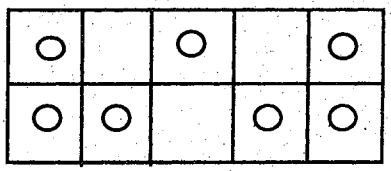


Natash

Natasha's representation for 20 is an example of how the brighter children were able to independently extend their own ideas to include number relationships beyond that traditionally taught at the kindergarten level. She shut her eyes and "saw two frames but I only had one frame to colour-in so I put two counters in each space and made twenty".



Patricia



Emily

Patricia's illustration did not immediately reveal any specific number relations and when questioned about her arrangement of dots it became evident that she did not understand the task and simply placed counters with no consideration to the arrangement of dots on the ten frame. An interview with Emily, on the other hand, revealed that she "saw" numerous part-part-whole relationships. Emily liked this arrangement because "there are 3 spaces and 7 dots and there are 3 here, 3 here and 1 in the middle. And there are 2 here and 2 here and 1, 2, 3 on top".

CONCLUSIONS AND IMPLICATIONS

The present study indicated that, with practice, children can become capable of mentally combining and separating patterns, thus able to instantly recognise the whole and its related parts - a very efficient strategy for aiding the recall of basic addition and subtraction facts. The ten frame imagery can be used as a visual reference in later work instead of relying on counting to derive sums of ten or answers to subtraction facts with minuends of ten. Although counting is certainly important, the objective of this paper is to stress the importance of developing visualising strategies as an efficient method for reinforcing number relations prior to the introduction of addition and subtraction in the symbolic form. For the majority of children in this study, it was found that once abstract mathematical symbols for the basic facts were finally introduced they were able to represent what they had previously visualised, talked about and illustrated in a very concise way.

More time needs to be devoted in the kindergarten and Year 1 mathematics programs to providing developmentally appropriate experiences that foster intuitive skills like visualisation. A program such as

that outlined in this paper has implications for exact and approximate mental arithmetic skills in later years.

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